Int. J. Heat Mass Transfer. Vol. 17. p. 378. Pergamon Press 1974. Printed in Great Britain

A COMMENT ON THE PAPERS "ON THE SOLIDIFICATION OF A WARM LIQUID FLOWING OVER A COLD WALL"

C. C. LII and M. N. ÖZISIK
Department of Mechanical and Aerospace Engineering, North Carolina State University, Raleigh, N. C. 27607, U.S.A.

(*Received* 28 *February* 1973 *and in revised form* 19 *July* 1973)

RECENTLY Elmas [1], in an attempt to develop an approximate solution to the solidification of a warm liquid flowing over a cold wall, reduced the energy equation into an integral form. Our examination of Elmas' analysis revealed that in the process of integration of the energy equation he left out the term $T(X, \theta)\mathrm{d}X/\mathrm{d}\theta$ from his equation (13), evidently by overlooking the fact that the transformed X co-ordinate was also a function of time. As a result, his equation (17) was in error and its correct form with his notation should be given as

$$\frac{\mathrm{d}}{\mathrm{d}\theta}\left\{\delta^2\left[\int_0^1 \eta T\mathrm{d}\eta - \tfrac{1}{2}\left(T_l + \frac{L}{C_p t_f}\right)\right]\right\} - \tfrac{1}{2}\frac{\mathrm{d}\delta^2}{\mathrm{d}\theta} \times \int_{\eta=0}^{1} \eta T\mathrm{d}\eta = T_w - 1 + (T_l - 1)Bi. \quad (1)$$

That is the term

$$\tfrac{1}{2}\frac{\mathrm{d}\delta^2}{\mathrm{d}\theta}\cdot\int_{\eta=0}^{1}\eta T\mathrm{d}\eta$$

is missing in his equation (17). This error was carried into equation (18) from which equation (21) in the integral form was derived for the thickness of the frozen layer. It was pointed out by Savino and Siegel [2] that the equation (21) could be integrated to yield a closed form solution of the same form as the second of four successive approximate solutions reported by them previously [3], but only with different coefficients. However, if the foregoing term were included in Elmas' analysis, the closed form solution to equation [21] given by Savino and Siegel [2], in their own notation, would be

$$\theta = \left[\tfrac{1}{4}(T_w + 1) - \left(T_l + \frac{L}{C_p t_f}\right)\right]\left[\frac{T_w - 1}{(T_l - 1)^2}\left(\frac{k}{h_l}\right)^2\right]\cdot \left\{\left(\frac{T_l - 1}{T_w - 1}\right)\frac{h_l\delta}{k} - \ln\left[1 + \left(\frac{T_l - 1}{T_w - 1}\right)\frac{h_l\delta}{k}\right]\right\}. \quad (2)$$

That is, the coefficient of the first term $(T_w + 1)$ is changed to $\frac{1}{4}$.

Finally, we would like to point out to the fact that equation (21) of [1], even in the corrected form, is based on the approximation of a uniform temperature distribution, therefore any solution obtained from that equation is also an approximate solution; whereas a more accurate treatment of the same problem using a more elaborate analysis is already given by Savino and Siegel [3].

REFERENCES

1. M. Elmas, On the solidification of a warm liquid flowing over a cold wall, *Int. J. Heat Mass Transfer* **13**, 1060–1062 (1970).
2. J. M. Savino and R. Siegel, A discussion of the paper "On the solidification of a warm liquid flowing over a cold wall", *Int. J. Heat Mass Transfer* **14**, 1225–1226 (1971).
3. J. M. Savino and R. Siegel, An analytical solution for solidification of a moving warm liquid onto an isothermal cold wall, *Int. J. Heat Mass Transfer* **12**, 803–809 (1969).